Should the cos (latitude) Coriolis terms be included in nonhydrostatic meso-scale operational forecast models?

Akira Kasahara, NCAR[1]

1. Overview

There are two nonhydrostatic dry dynamics effects: One is due to the vertical acceleration term, and the other is due to the cos (latitude) Coriolis terms. The need of including the vertical acceleration for meso-scale forecasts is hardly an issue, since the consequence of neglecting the vertical acceleration in prediction models is well understood in all coordinate systems, including midlatitude and equatorial beta-planes (Qian and Kasahara, 2003) and in spherical geometry (Kasahara and Qian, 2000). In contrast, although considerable progress has recently been made in understanding the role of cos (lat) Coriolis terms, the dynamical effects of these terms are still elusive. In fact, the inclusion of these terms in nonhydrostatic models without careful consideration could have detrimental impacts on forecasts in somewhat lesser extent as we experienced during 1960's to 70's in the transition from the use of quasi-geostrophic to primitive-equation models. It seems that the need of understanding the role of cos (lat) Coriolis terms in numerical models is urgent.

2. Normal modes of compressible and stratified model

It has been known that the normal modes of compressible and stratified model on Cartesian tangent-plane with the sin (latitude) Coriolis terms consists of two pairs of acoustic and inertio-gravity internal modes and one pair of external (Lamb) modes. When the model also includes the cos (latitude) Coriolis terms and the model is bounded by top and bottom rigid boundaries, there emerges a pair of extra normal modes which are called "new modes" by Thuburn et al. (2002b) and "boundary-induced inertial (BII) modes" by Kasahara (2003 a,b).

The perturbed vertical velocity $w(x, y, z, t)$ of the linearized model, given in Section 4 of Kasahara (2003a), is expressed by

$$w = W(z) \exp[i(mx + ny - \sigma t)], \tag{1}$$

where m, n, and σ denote, respectively, the zonal and meridional wavenumbers and the frequency, and $i = \sqrt{-1}$. Note that the vertical function, $W(z)$, is bounded by

$$W(z = z_T) = 0 \quad \text{and} \quad W(z = 0) = 0. \tag{2}$$

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The solutions of the vertical equation for $W(z)$ are given by

$$W(z) = \sin(kz)\exp(i\Gamma_2 z), \tag{3}$$

where k denotes the vertical wavenumber and the parameter Γ_2 is expressed by

$$\Gamma_2 = -\frac{f_H f_V n}{f_V^2 - \sigma^2} \quad . \tag{4}$$

Here $f_V = 2\Omega \sin\phi$ and $f_H = 2\Omega\cos\phi$, where ϕ is the latitude and Ω the angular rotation rate of the earth.

As mentioned earlier, we find two pairs of frequencies corresponding to acoustic (σ_a) and inertio-gravity (σ_g) modes, as well as one pair of the external (Lamb) modes. In addition, there is one pair of BII (σ_I) modes which have near-inertial frequencies. If the thermal stratification factor N (Brunt-Väisälä frequency) is much larger than the rotation rate Ω, we find

$$\sigma_I^2 \doteq f_V^2 \left[1 - \frac{n^2 f_H^2}{(m^2+n^2)N^2}\right] \quad . \tag{5}$$

For a historical discussion on derivation of the above approximate solutions, the reader is referred to Kasahara (2004b).

The presence of the factor $\exp(i\Gamma_2 z)$ in (3) is the unique aspect of this model when the f_H-terms are included. Here, we estimate the magnitude of Γ_2, which has the dimension of vertical wavenumber, for the BII modes, using (5) as follows:

$$|\Gamma_2| = \frac{(m^2+n^2)N^2}{n f_H f_V} = \frac{2\pi}{D}, \tag{6}$$

where we introduce the vertical scale D to represent the vertical wavenumber for Γ_2. By assuming that $m \sim n = 2\pi/L$, where L denotes the horizontal scale of motion, we obtain the aspect ratio D/L that can be expressed as

$$\frac{D}{L} = \frac{f_H f_V}{2N^2}. \tag{7}$$

By choosing that $f_H \sim f_V = 10^{-4}\mathrm{sec}^{-1}$, the value of D/L becomes 0.5×10^{-4} for $N = 10^{-2}\ \mathrm{sec}^{-1}$ and 0.5×10^{-2} for N=$10^{-3}\ \mathrm{sec}^{-1}$. Thus, in this range of N that is appropriate for the oceans and the atmosphere, the value of D ranges from 5m to 500m for L=100km. This means that the vertical structure of the BII modes has a rather high variability modulated by the $\sin(kz)$ term. The same observation was made by Thuburn et al. (2002b, Fig. 7) who illustrated the eigenstructures of the zonal wind component and the pressure which are strongly tilted with a very high vertical variability of "typically a few meters to a few hundred meters" in vertical wavelength.

In contrast, for σ_g^2 the magnitude of Γ_2 becomes extremely small. A similar analysis shows that the aspect ratio D/L in this case is estimated by

$$\frac{D}{L} = \frac{N^2}{f_H f_V}, \tag{8}$$

using an approximation of σ_g^2 by N^2. Thus, in the range of N from 10^{-2} sec^{-1} to 10^{-3} sec^{-1}, the value of D ranges from 10^9m to 10^7m for $L=$ 100km. Therefore, the factor exp $(i\Gamma_2 z)$ is practically unity and can be neglected for the σ_g modes.

3. Discussions

(1) One unique dynamical feature in the nonhydrostatic model on Cartesian tangent-plane with inclusion of cos (lat) Coriolis terms is the presence of boundary-induced inertial (BII) modes which could have a high variability in the vertical direction. The presence of BII modes has been confirmed by an analytical consideration. One question related to the BII modes is whether such dynamical feature has been observed in reality. Kasahara (2004b) discussed this issue and speculated that the well-known near-inertial oscillations, which are dominant and ubiquitous though intermittent, in the oceans can possibly be explained by combined actions of the inertio-gravity and BII-mode oscillations.

(2) Another question is whether the BII modes can be found in a finite-difference model in which the stratification parameter, N, is variable in height. The normal mode analysis can be carried out in a discretized domain by using a matrix formulation. However, the likelihood of detecting the BII modes in a discretized model may depend on details of grid discretization and how to solve matrix problems. Work is in progress and findings will be reported later.

(3) The presence of BII modes can likely be detected in discretized numerical models by the initial-value approach. So far, we are aware of only one reference that may hint at the presence of BII modes in an enclosed domain. Beckmann and Diebels (1994) conducted a numerical experiment using an enclosed ocean model and calculating eigen-oscillations of the internal response to a single pulse and forced seamount trapped waves. They found rather unique dynamical effects of cos (lat) Coriolis terms that are likely explained from the normal mode analysis of the corresponding model.

(4) It has been said that the cos (lat) Coriolis effects become only important when the stratification parameter N is small compared with the Earth's rotation rate. This statement refers only to the inertio-gravity modes and does not apply to the BII modes which appear regardless of the magnitude of N as long as the boundary constraints exist. As the limit of N goes to zero, the BII-mode frequencies become larger than the Coriolis frequency. Thus, two kinds of eigenfrequencies produce a spectrum of discrete frequencies, spanning from 2Ω to -2Ω for various combinations of zonal, meridional, and vertical wavenumbers.

(5) Lastly, a comment should be made on the likelihood of having similar modes as the BII modes in a compressible, stratified, nonhydrostatic, deep atmospheric model in spherical geometry. Thuburn et al. (2002a) calculated the normal modes of the global model using a finite-difference method in the meridional and radial directions, while harmonic solutions are assumed in the longitude. Kasahara (2004a) also investigated the problem using a spherical harmonic expansion in the horizontal and a finite-difference scheme in the radial direction. However, both studies have shown that the BII-type modes are lacking. It is beyond the scope of the present talk to discuss this issue.

4. References

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